Electromagnetic Education – Development of Models and Measurement Validation

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The study of electromagnetics is of fundamental importance to engineers working with RF systems. Many engineers find the theory and application of the theory to be challenging. Importantly, engineers need insight into the development of good analytical models require subsequent validation with experimental measurements. The challenge at the academic level is to teach the theory in such a way that leads to the needed insight. This insight can often be developed with good case studies.

It is also noted that a good background in electromagnetic theory necessarily includes an understanding of measurement techniques. Armed with such knowledge, an engineer has the background to question the accuracy of the measured data given disagreement with analytical predictions.

An academic case study compares electromagnetic models to data considers the measurement of the radiation pattern of an antenna in the far-field (i.e., in the Fraunhofer zone). Engineers are taught criteria that ensures far-field measurements. For example, the distance to the far-field is generally presumed to be given by the Raleigh condition: $R > 2D^2/\lambda$.

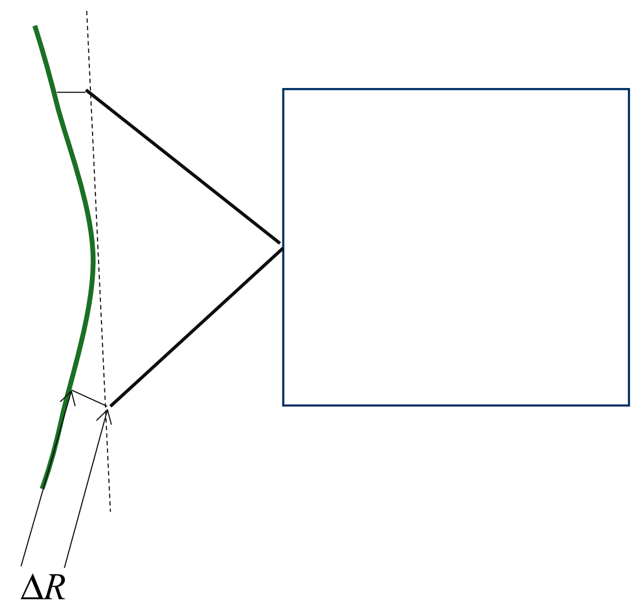
For this condition, D is the largest linear dimension of the antenna while λ is the wavelength of the carrier frequency. This far-field criteria is not suitable for the measurement of antennas with ultra-low sidelobes. In this case, the analytical models may predict sidelobe levels that exist but require more measurement distance to be detectable.

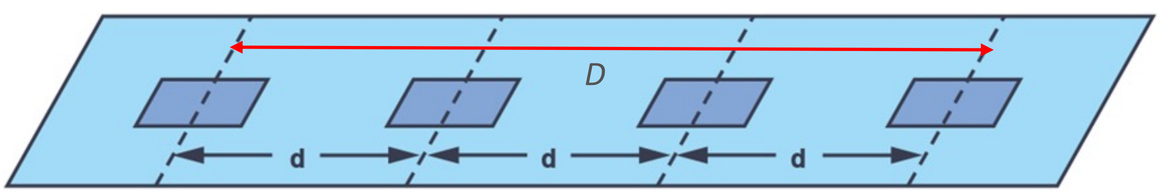
Electrically small antennas (small relative to wavelength) require knowledge of other far-field criteria (i.e., the geometrical and electrical conditions). Again, an RF engineer need a proper understanding of the associated electromagnetic theory and definitions before proceeding to measurements.

This presentation will examine this and other case studies and emphasize how they can be used to increase the quality of metrology in the university academic curricula.

Spherical Waves and Phase Errors

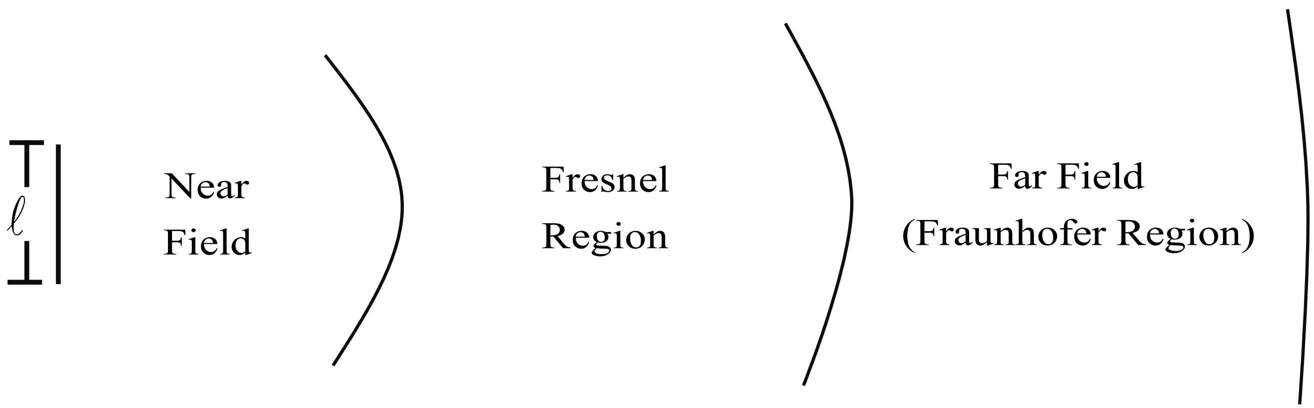
A spherical wave is shown impinging on the aperture of the antenna. Note that there is some distance "d" that exists across the extremes of the aperture that the wave must still travel. This distance translates into a phase error. $\psi_{error} = k\,d$. Clearly, $\psi_{error} = 0$ if $d = 0$ and we have a plane wave over the extent of the aperture. Antenna theory assumes a plane wave incident on the antenna. In reality, there is always some curvature do to the incoming spherical wave. An acceptable maximum phase error it typically specified as $\psi_{error} = k\,d = \dfrac{\pi}{8} = 22.5^{\circ}$. From this specified error, we define the far-field (Fraunhofer Region) at a distance of $R \geq \dfrac{2D^2}{\lambda}$. However, this distance may not be suitable for measuring low sidelobe levels.





https://www.analog.com/en/resources/analog-dialogue/articles/phased-array-antenna-patterns-part1.html

This analysis applies to electromagnetically long antennas where the length of the antenna "ℓ" and wavelength are such that: $r \gg \ell$. and $r \gg \lambda$ respectively.



Our field regions are shown in the figure above. Using a line current as the standard, we identify three regions: The near field, Fresnel region, and the far field. The far field is the region we normally operate in with our antenna. As a review, recall that the free-space Green's function is asymptotically represented as:

$$\frac{e^{-jk\left|\vec{r}-\vec{r}'\right|}}{\left|\vec{r}-\vec{r}'\right|} \approx \frac{e^{-jkr}}{r} e^{+jk\hat{a}_r \bullet \vec{r}'} \qquad \left|\vec{r}\right| \gg \left|\vec{r}'\right|$$

Evolution of Pattern from Near to Far Field

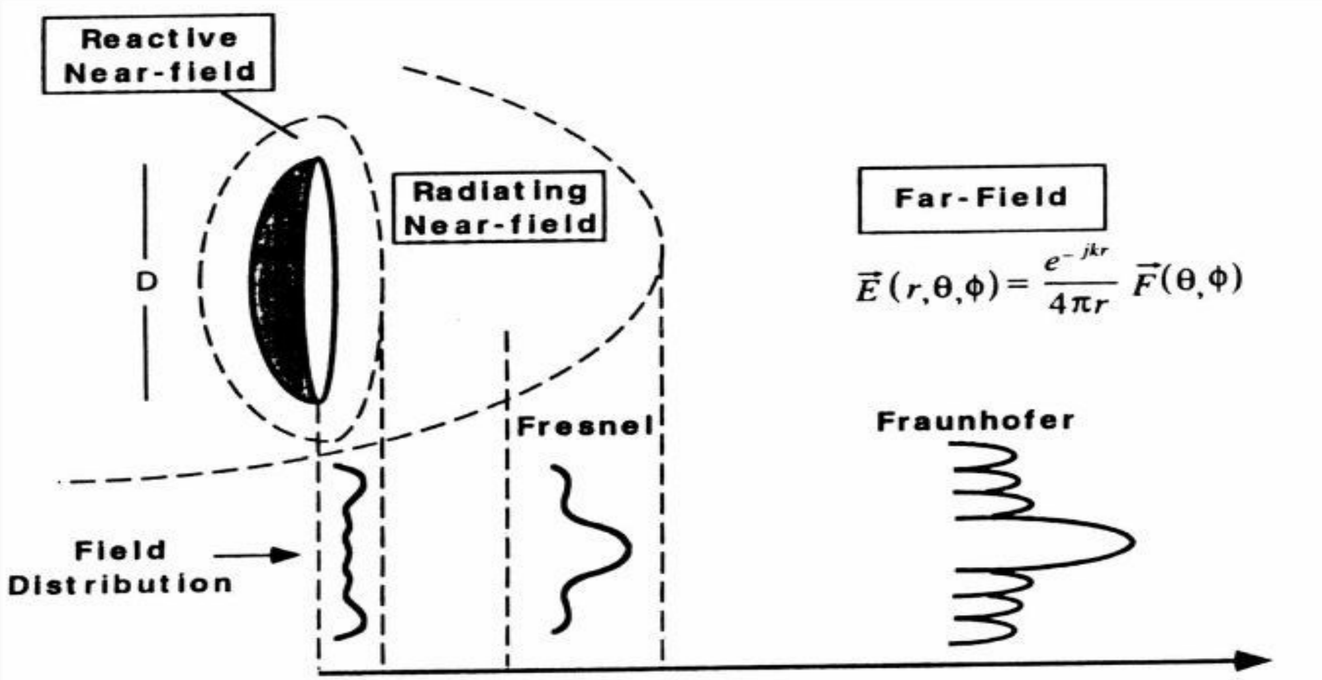


Fig. 2.8

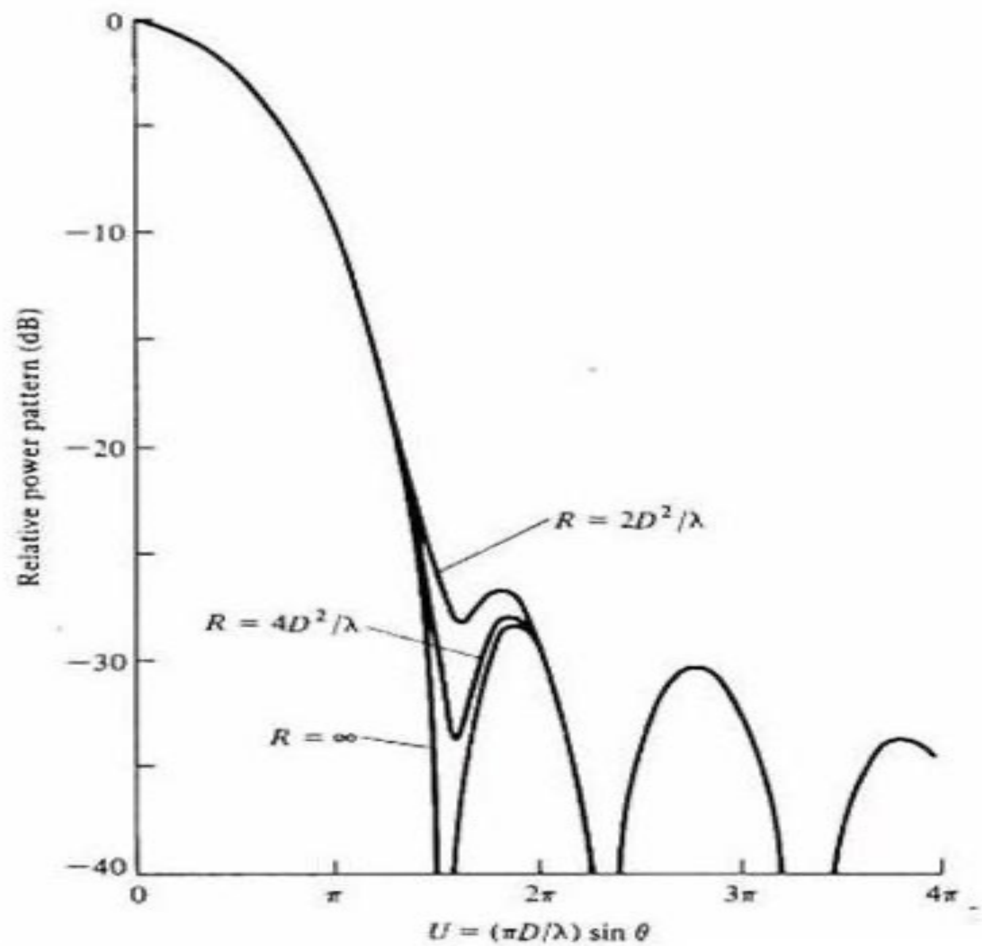
$$\frac{e^{-jk\left|\vec{r}-\vec{r}'\right|}}{\left|\vec{r}-\vec{r}'\right|} \approx \frac{e^{-jkr}}{r}\; e^{+jk\left(z'\cos(\theta) - \frac{1}{r}\frac{z'^{2}}{2}\sin^{2}(\theta) - \frac{1}{r^{2}}\frac{z'^{3}}{2}\cos(\theta)\sin^{2}(\theta) + \ldots\ldots\right)}$$

Alignment of a uniform distribution along the z-axis. Keeping only first term in the second term in the second exponential with a phase error of 22.5 Degrees results in criteria $R = 2D^2/\lambda$.

Spherical Waves and Phase Errors

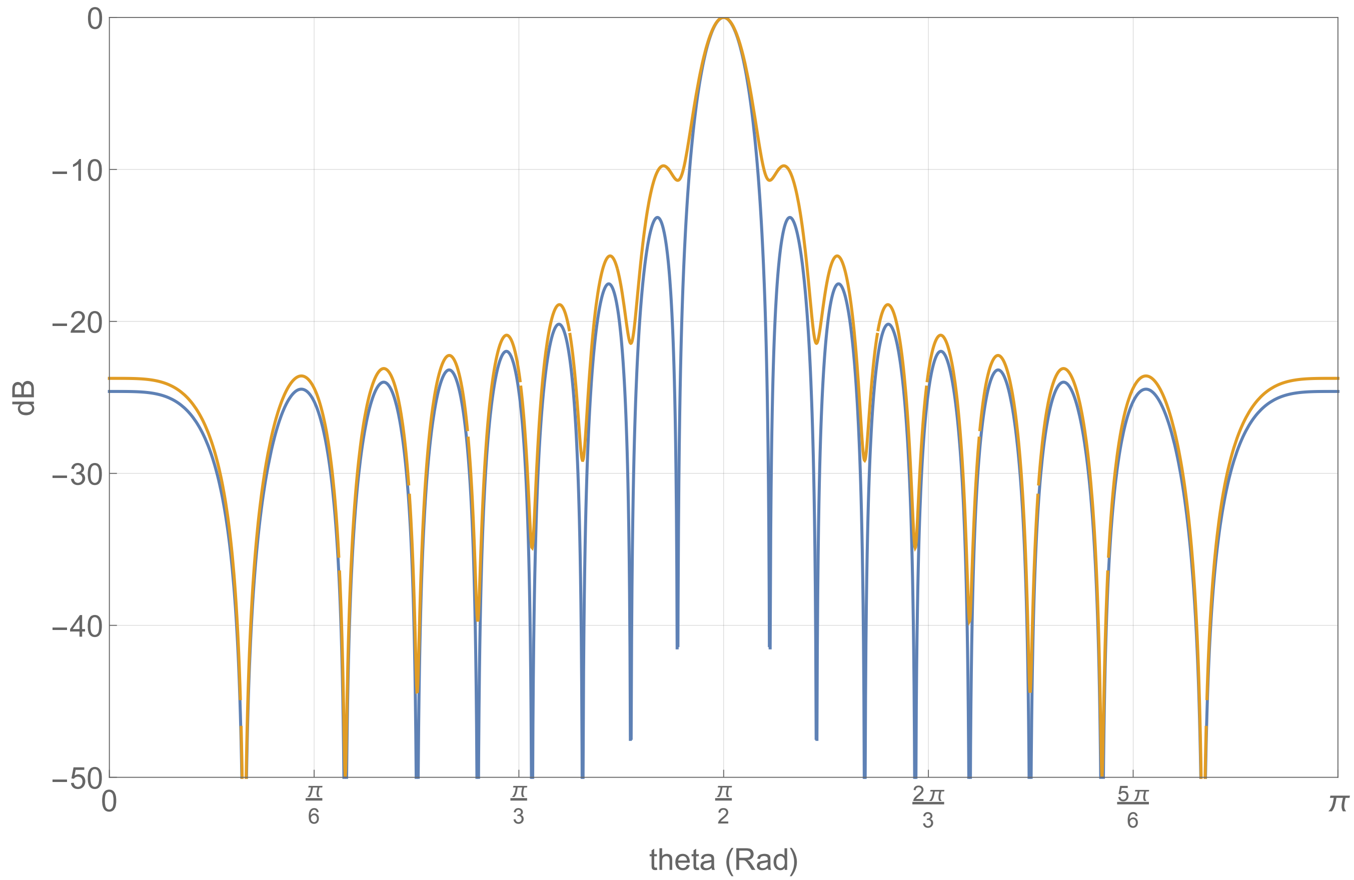
Maximum phase error at the edge of the aperture

Distance	Radians	Degrees	Wavelength
Infinity	0	0	0
$32\ D^2 / \lambda$	$\pi/128$	1.406	$\lambda/256$
$16\ D^2 / \lambda$	$\pi/64$	2.813	$\lambda/128$
$8\ D^2 / \lambda$	$\pi/32$	5.625	$\lambda/64$
$4\ D^2 / \lambda$	$\pi/16$	11.250	$\lambda/32$
$2\ D^2 / \lambda$	$\pi/8$	22.500	$\lambda/16$ ← Frequently used
D^2 / λ	$\pi/4$	45.000	$\lambda/8$
$0.5\ D^2 / \lambda$	$\pi/2$	90.000	$\lambda/4$
$0.25\ D^2 / \lambda$	π	180.00	$\lambda/2$


0
−10
−20
−30
−40
Relative power pattern (dB)
R = 2D²/λ
R = 4D²/λ
R = ∞
0
π
2π
3π
4π
U = (πD/λ) sin θ

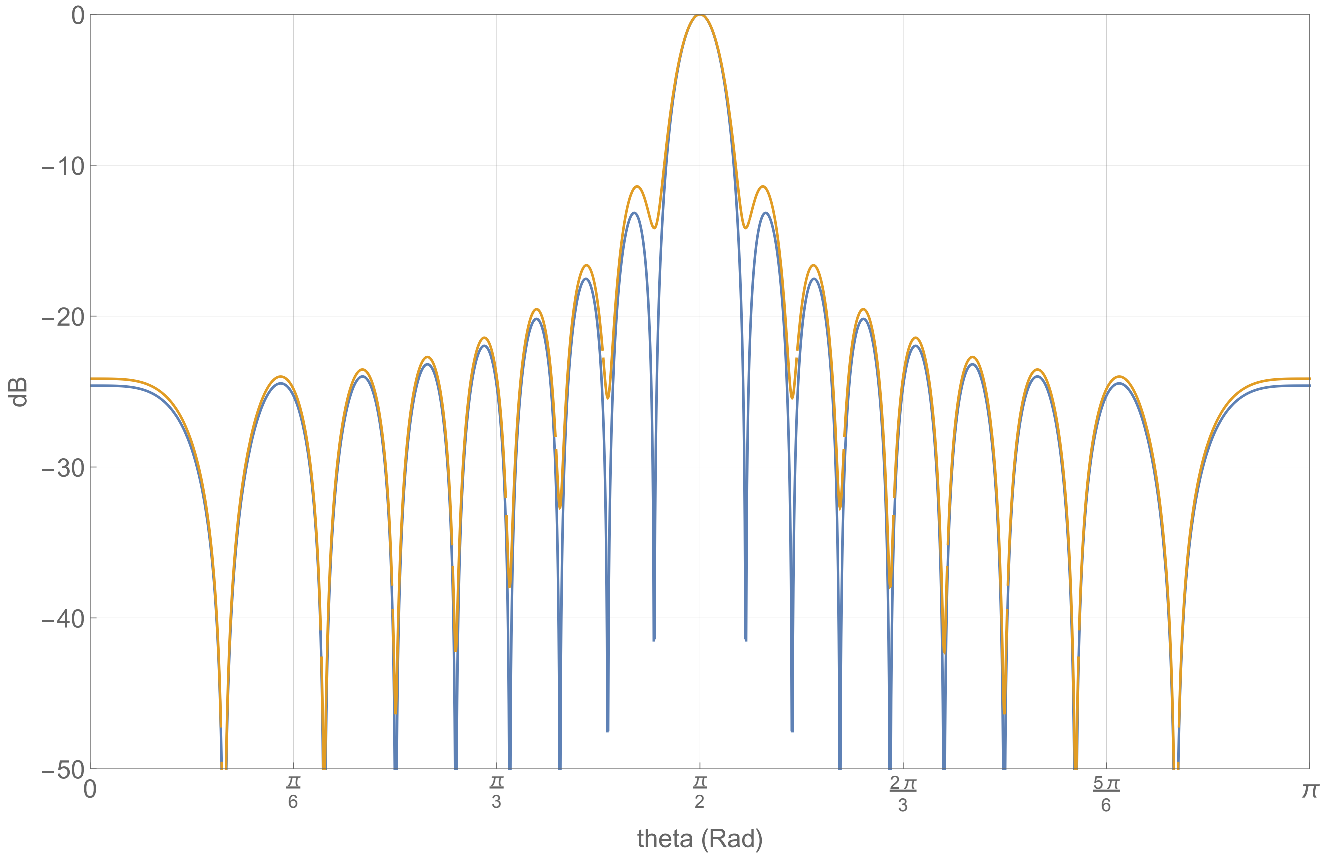
Uniform Array (17 Elements)

Blue line - no phase error. Orange line 22.5 Degree phase error. $R = 2D^2/\lambda$.



Uniform Array (17 Elements)

Blue line - no phase error.   Orange line 22.5 Degree phase error. $R = 2D^2/\lambda$.

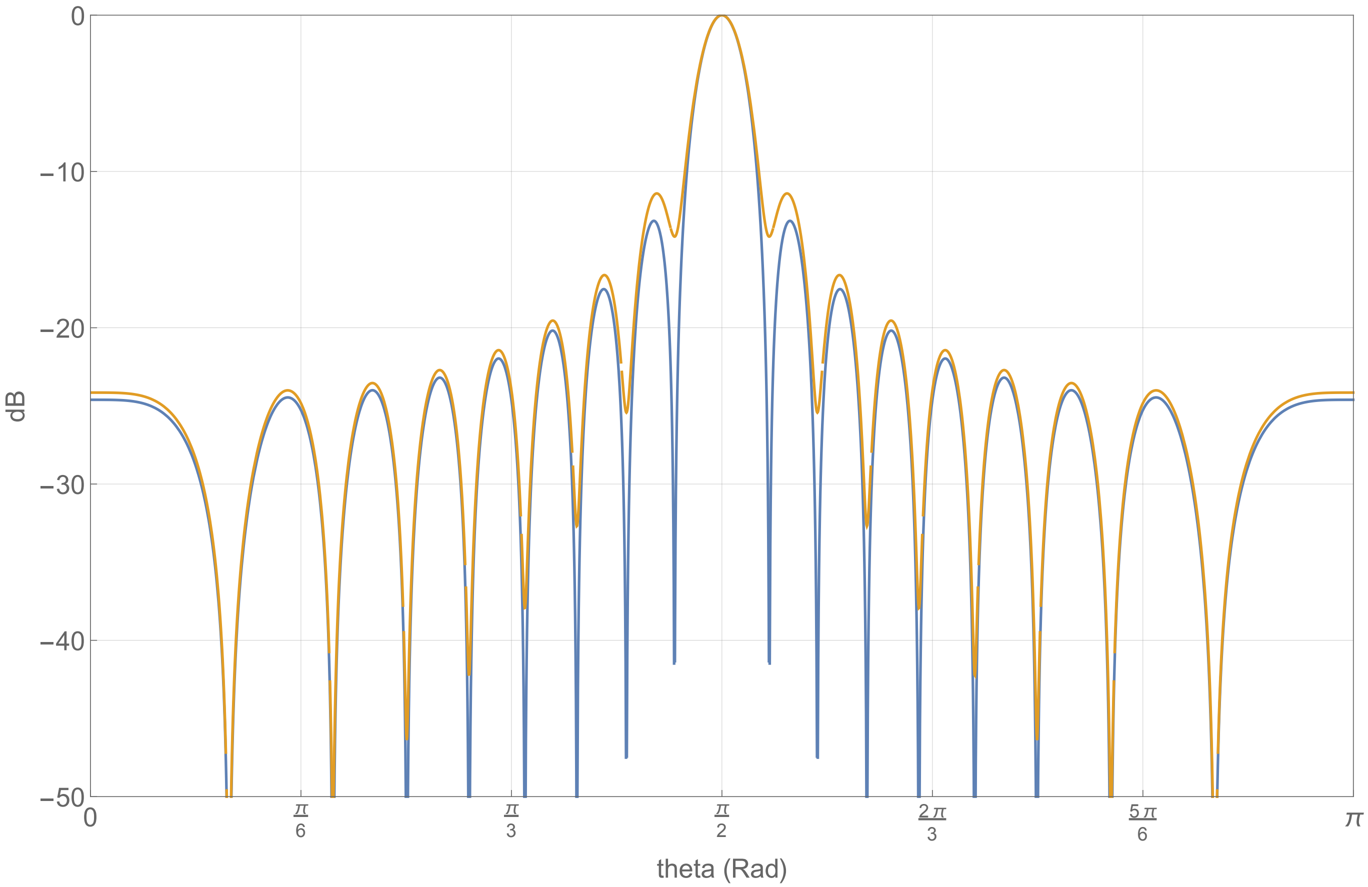


$$e^{+jk(z'\cos(\theta) - \frac{1}{r}\frac{z'^2}{2}\sin^2(\theta))}$$

Correction term included.

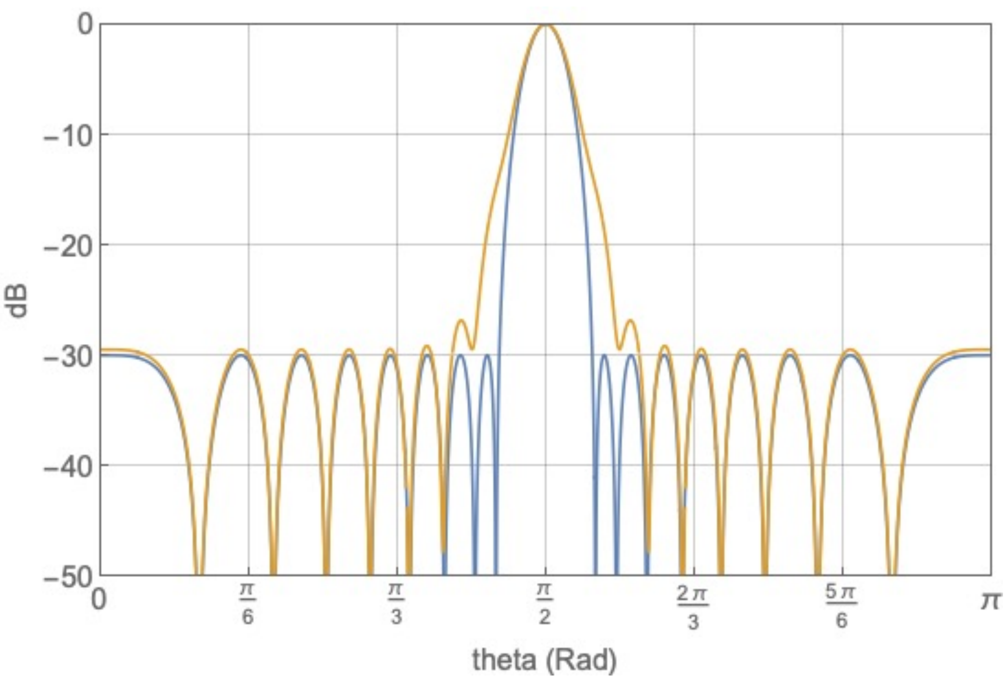
Uniform Array (17 Elements)

Blue line - no phase error. Orange line 11.25 Degree phase error. $R = 4D^2/\lambda$.

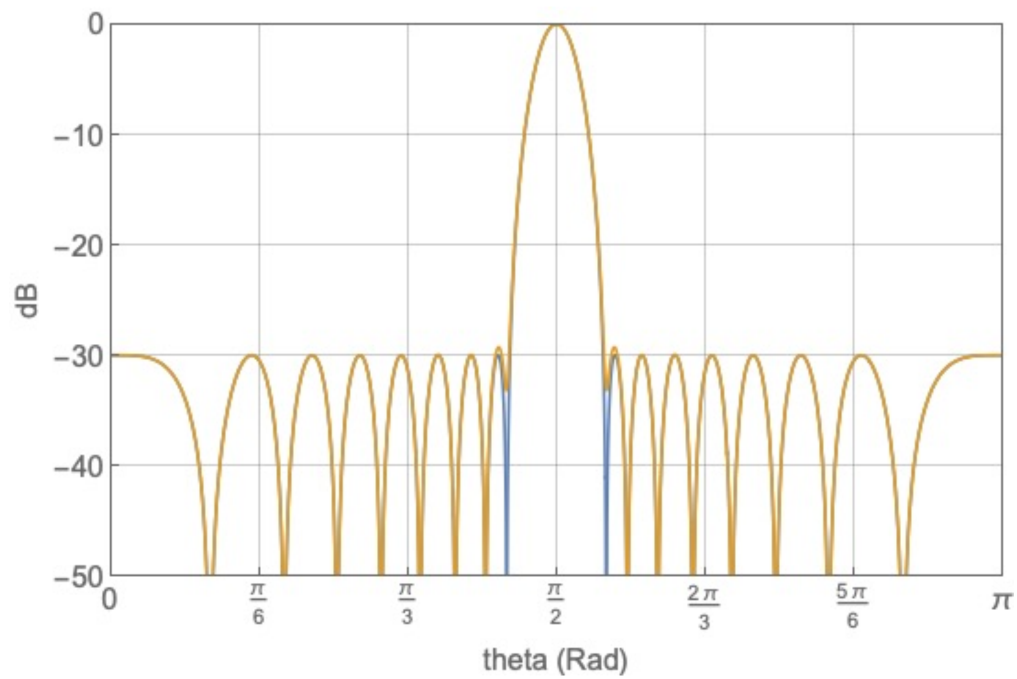


Chebyscheff Array (17 Elements) with 30 dB specified sidelobe levels

Orange line 22.5 Degree phase error. $R = 2\,D^2/\lambda$.



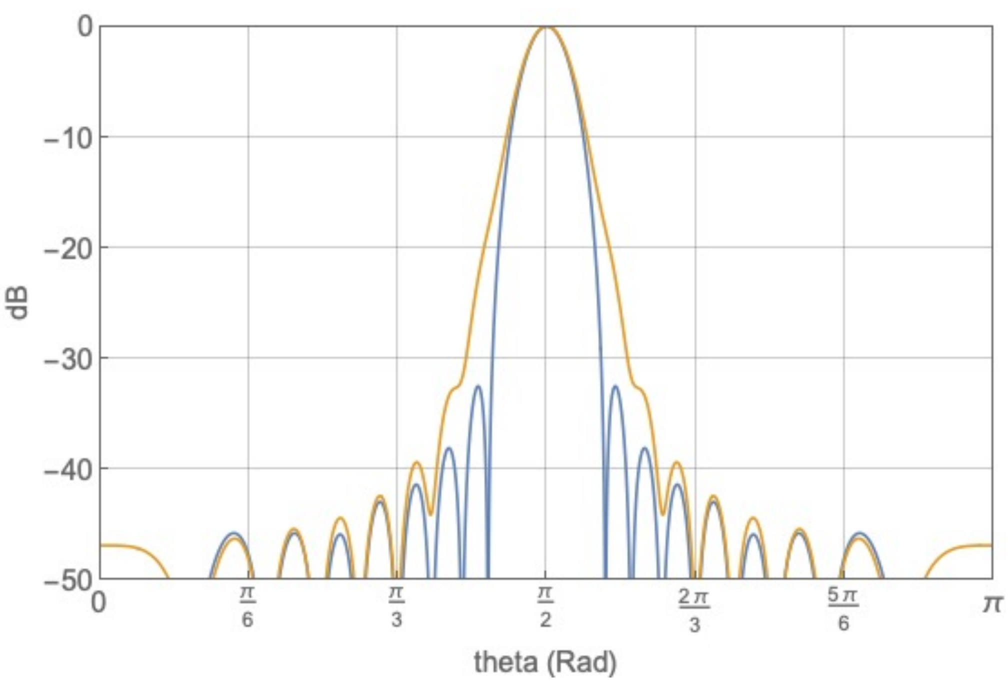
Orange line 2.813 Degree phase error. $R = 16\,D^2/\lambda$.



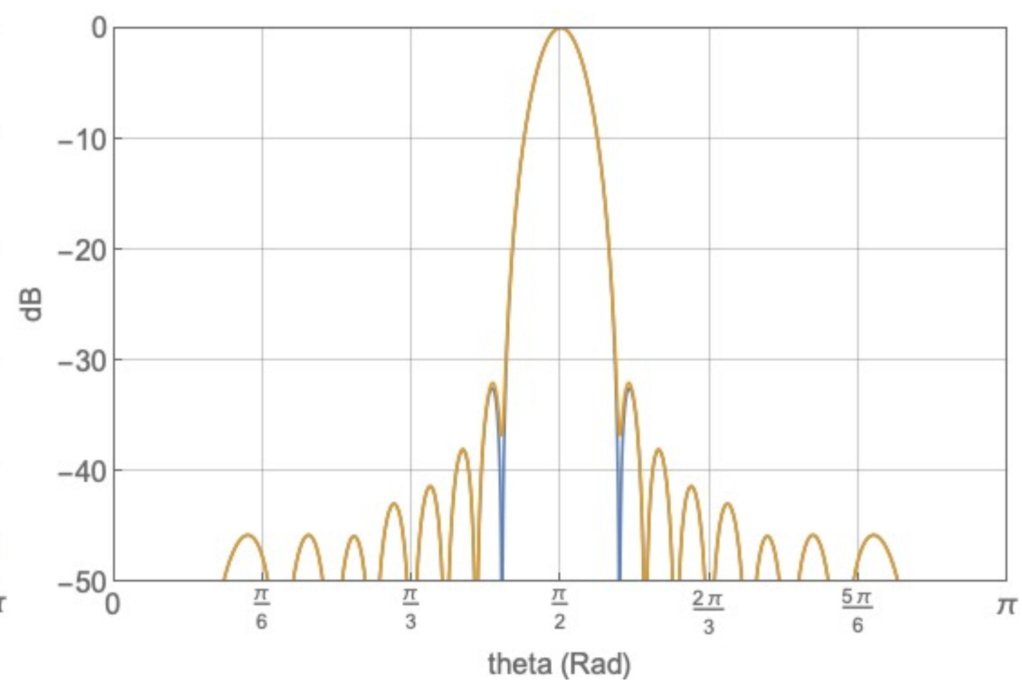
Blue line - no phase error.

Taylor Array (17 Elements) with 30 dB specified sidelobe levels

Orange line 22.5 Degree phase error. $R = 2\,D^2/\lambda.$

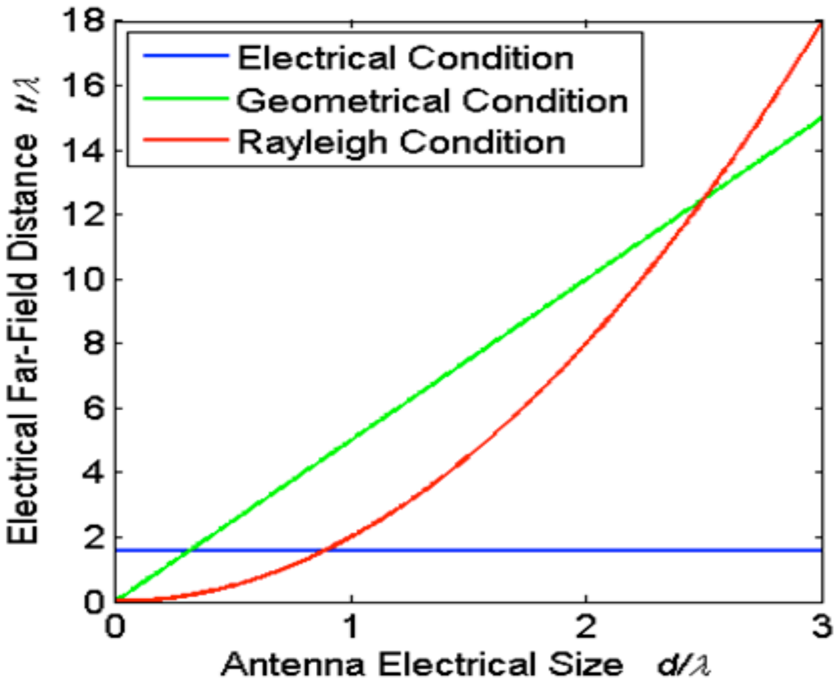


Orange line 2.813 Degree phase error. $R = 16\,D^2/\lambda.$



Blue line no phase error.

Far-field conditions. The electrical distance to the observation point r/λ as function of the electrical size d/λ of the antenna for the electric far-field condition, the geometrical far-field condition, and the Rayleigh far-field condition. Depending on the electrical size of the antenna (d/λ) any of the three conditions may be the stricter one defining the far-field region.



Electrical Condition : $r/\lambda > 5/\pi$ Blue

Geometrical Condition : $r/\lambda > 5d/\lambda$ Green

Rayleigh Condition : $r/\lambda > 2(d/\lambda)^2$ Red

Far Field of an Electrically Small Antenna

Summary of far-field conditions: $r > 2D^2/\lambda$, $r > 1.6\lambda$, $r > 5D$

Region	Distance from antenna (r)
Reactive near field	0 to $0.62\sqrt{D^3/\lambda}$
Radiating near field	$0.62\sqrt{D^3/\lambda}$ to $2D^2/\lambda$
Far Field	$2D^2/\lambda$ to ∞

Example: A small dipole of length $\lambda/50$.

$r > 2D^2/\lambda$ implies $2(\lambda/50)^2/\lambda = 0.0008\,\lambda$ (no - Rayleigh condition)

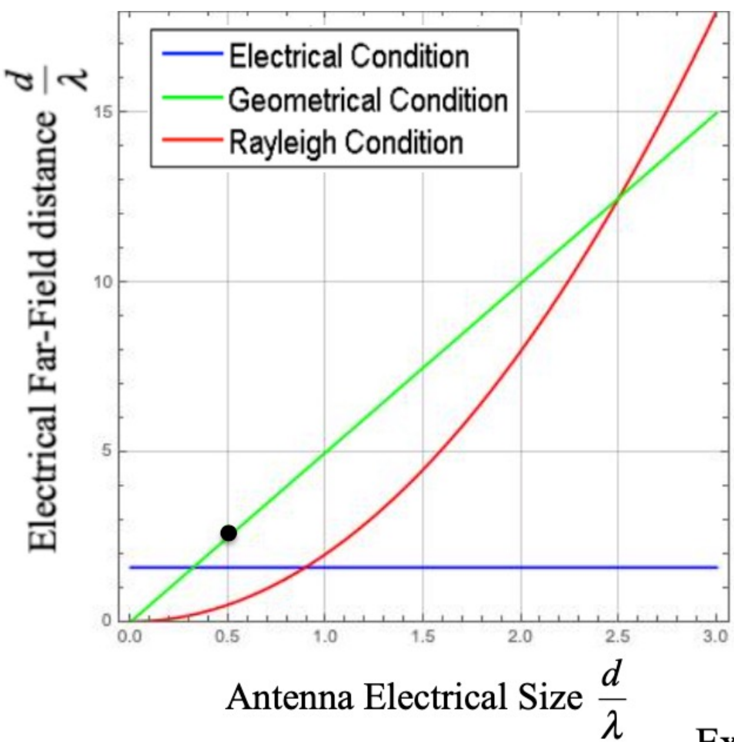
$r > 5D$ implies $(5)\lambda/50 = 0.1\,\lambda$ (no - Geometrical Condition)

$r > \frac{5}{\pi}\lambda$ implies $(1.6)\lambda$ (yes - Electrical Condition is the largest!)

Be careful when determining the far field with electrically small antennas!

Usually, the $2D^2/\lambda$ does not put you into the far field.

Far Field of a Half-Wave Dipole

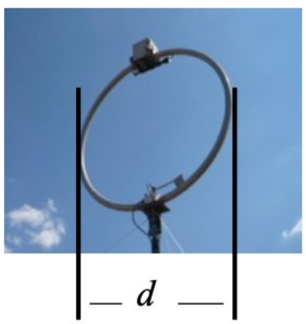


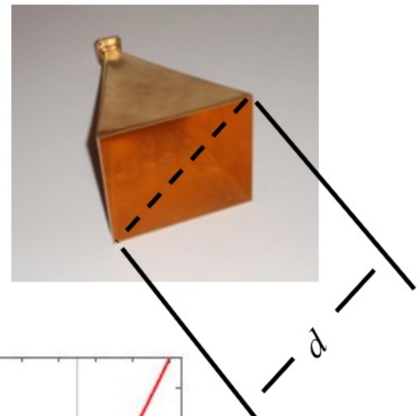
Example: A half-wave dipole of length $\lambda / 2$.

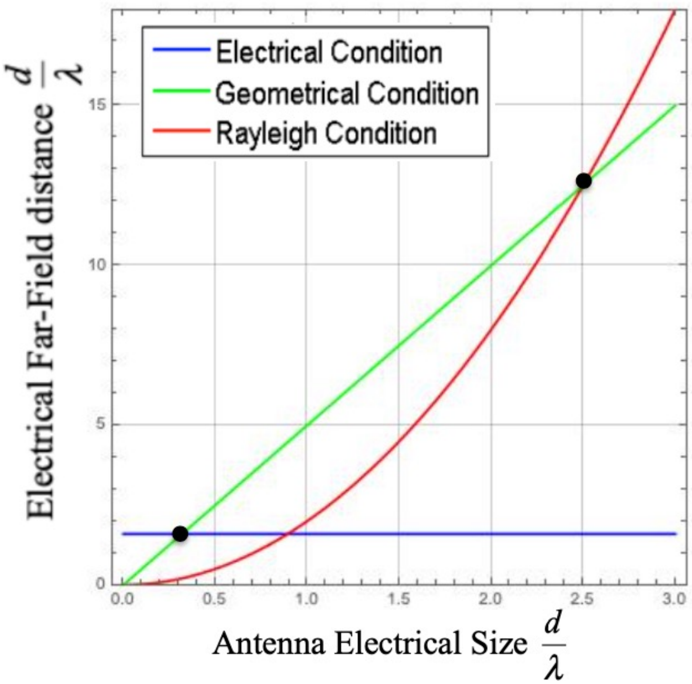
$r > 2D^2/\lambda$ implies $2(\lambda/2)^2/\lambda = 0.5\,\lambda$ (no - Rayleigh condition)

$r > 5D$ implies $(5)\lambda/2 = 2.5\,\lambda$ (Yes - Geometrical Condition)

$r > \frac{5}{\pi}\lambda$ implies $(1.6)\lambda$ (no - Electrical Condition)

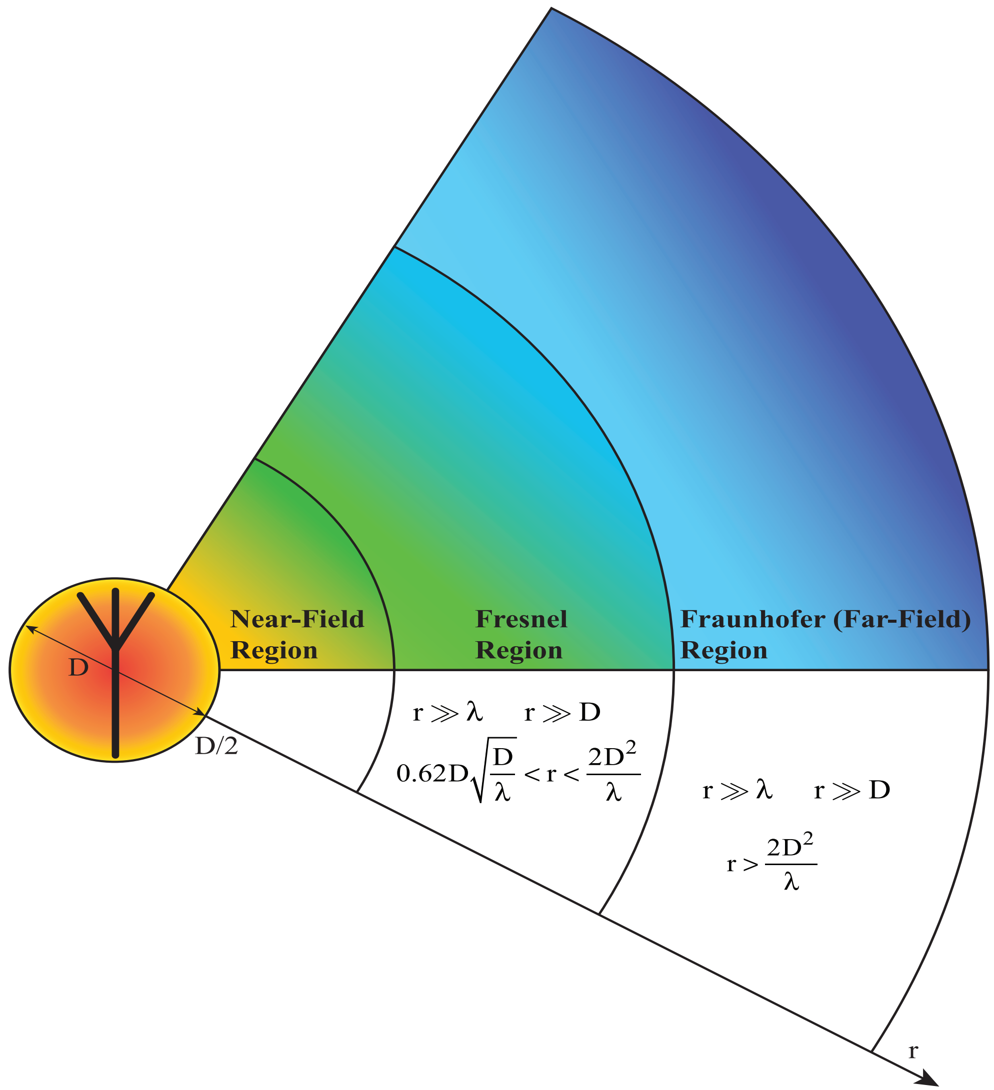






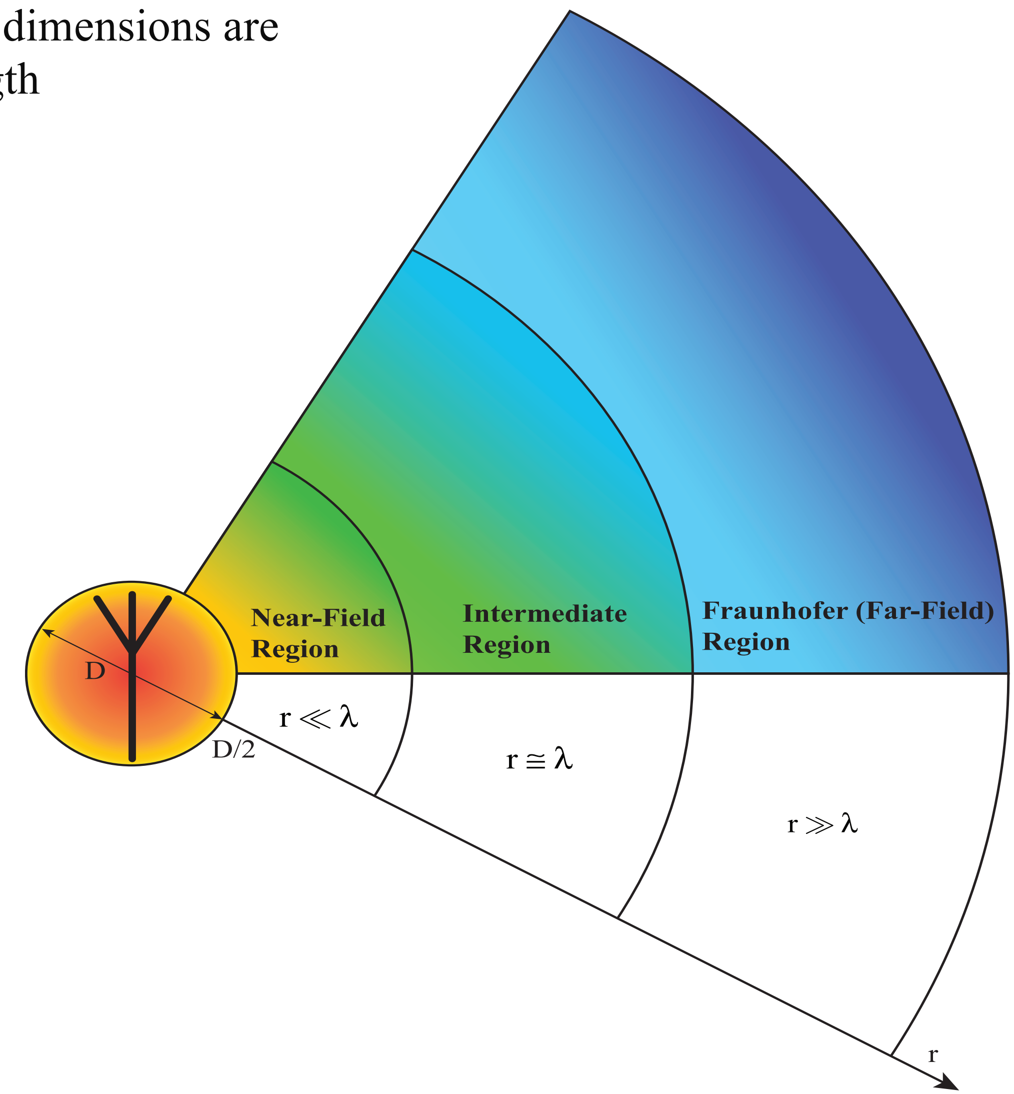
The Rayleigh condition is used when $\frac{d}{\lambda} > 2.5$ and we are greater than 12.5 wavelengths from the antenna.

We use the geometrical condition when: $0.318 \leq \frac{d}{\lambda} \leq 2.5$.



Near-Field Region
Fresnel Region
Fraunhofer (Far-Field) Region
D
D/2
$r \gg \lambda \quad r \gg D$
$0.62D\sqrt{\frac{D}{\lambda}} < r < \frac{2D^2}{\lambda}$
$r \gg \lambda \quad r \gg D$
$r > \frac{2D^2}{\lambda}$
r

Field Regions for antennas whose dimensions are small with respect to the wavelength



References

IEEE Standard for Definitions of Terms for Antennas

IEEE Antennas and Propagation Society

Sponsored by the

Antennas Committee

IEEE
3 Park Avenue
New York, NY 10016-5997
USA

IEEE Std 145™-2013
(Revision of
IEEE Std 145-1993)

References

1) C. A. Balanis, *Antenna Theory: Analysis and Design*, 4th ed. Wiley, 2016.

2) W. L. Stutzman, G. A. Thiel, *Antenna Theory and Design*, 3rd ed., Wiley, 2013.

3) IEEE Standard for Definitions of Terms for Antennas, IEEE Std 145-2013, December 2013.