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Teaching the Methods for the Calculation of the Resonant Modes of Patch Antennas for Comparison to Simulated and Measured Data

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Overview

In a classroom environment, the resonant modes of patch antennas are frequently taught using the cavity model as a baseline. Calculation of the various resonant frequencies are easily found presuming a TE/TM decomposition of the field between the patch and the underlying ground plane while assuming fictitious magnetic walls about the perimeter of the antenna. This model provided a good first-order estimate of the frequencies for the resonant modes. However, the cavity model neglects the fringing fields at the edges of the patch and corrections are often suggested for more refined calculations. These refinements take the form of an effective dielectric constant and a calculation of effective length that essentially shortens the patch by a length that takes into account the substrate thickness. Students are taught these models and given the equations for making the corrections. In this presentation, the value of such refinements will be discussed, and the results compared to simulation and measurement.

Resonant Frequency with the Cavity Model

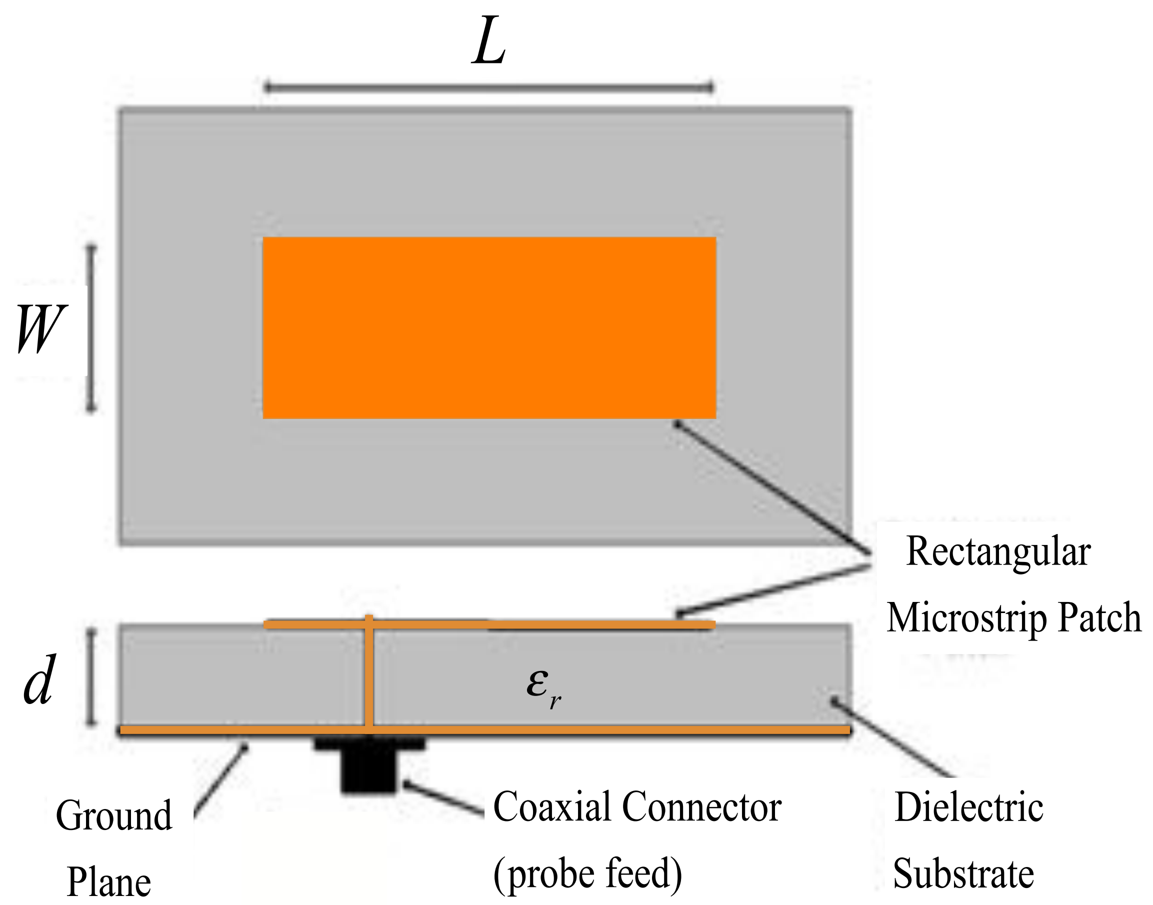
Resonant frequency as found using the cavity model:

$$f_r = \frac{c}{2\pi\varepsilon_r}\sqrt{\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{W}\right)^2}$$

For the lowest resonant mode

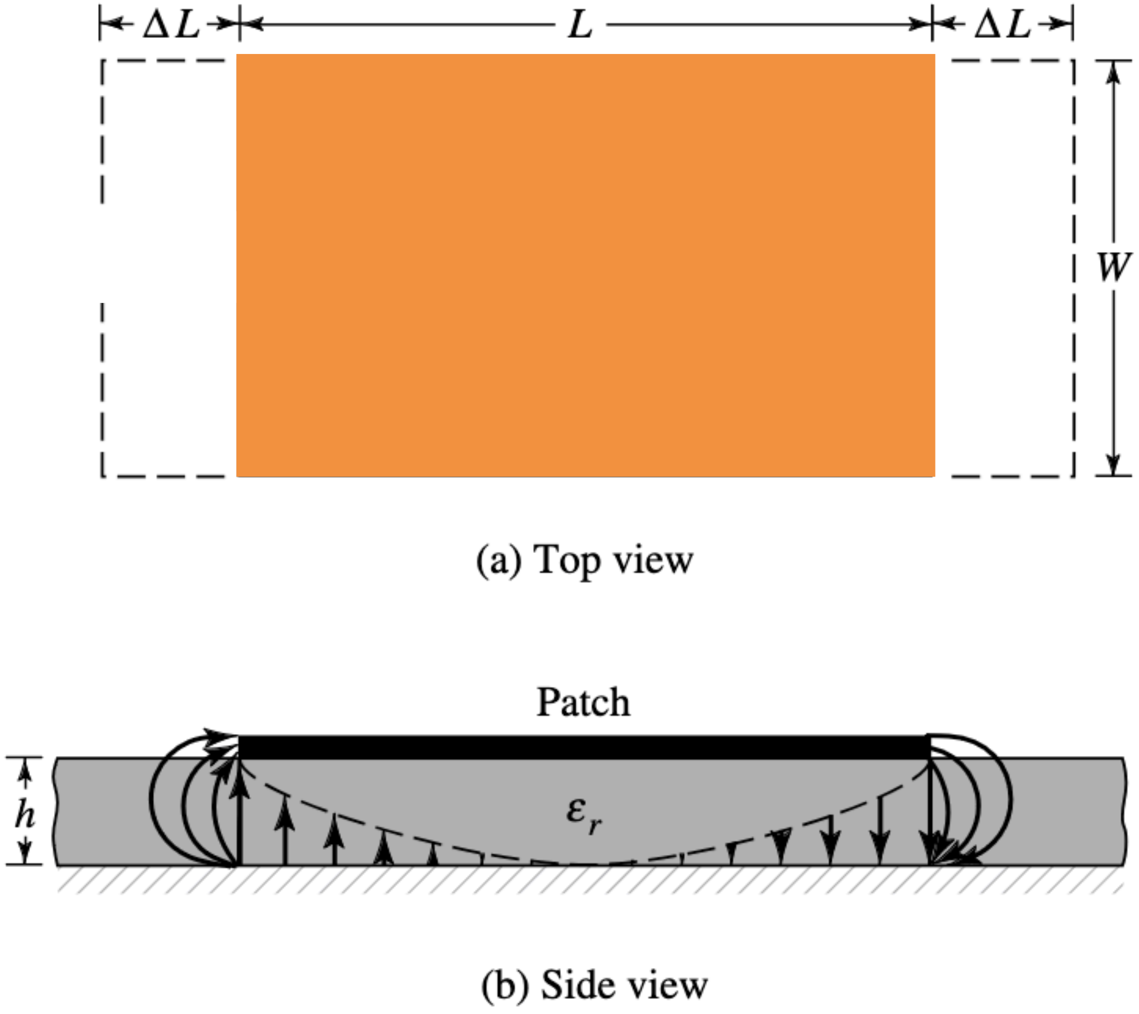
where $L > W$:

$$f_r = \frac{c}{2\varepsilon_r L}$$



Resonant Frequency Refinements

Fringing effects increase the resonant wavelength underneath the patch. Accordingly, the cavity model will always predict a resonance higher that will be measured. Refined procedures subtract some length from each side ΔL. Additional refinements replace the dielectric constant ε_r with an effective dielectric constant ε_{reff}.



Adjustments

Equations used to refine the cavity model:

1) $\varepsilon_{reff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{W}\right]^{-1/2}$ [1]

2) $\Delta L = 0.412\,h\frac{(\varepsilon_{reff} + 0.3)\left(\frac{W}{h} + 0.264\right)}{(\varepsilon_{reff} - 0.258)\left(\frac{W}{h} + 0.8\right)}$ [2]

3) $L_{eff}(\varepsilon_{reff}) = L + 2\Delta L$

4)3) $L_{eff}(\varepsilon_r) = L + 2\Delta L$ replaces ε_{reff} with ε_r in equation 2.

[1] C. A. Balanis, Advanced Engineering Electromagnetics, Second Edition, John Wiley & Sons, New York, 2012.
[2] E.O. Hammerstad, "Equations for Microstrip Circuit Design," *Proc. Fifth European Microwave Conference*, PP. 2668-272, Sept. 1975.

Adjustments

Next, we present calculations of the resonant frequency for various refinements

Cavity model: $f_r = \dfrac{c}{2\varepsilon_r L}$

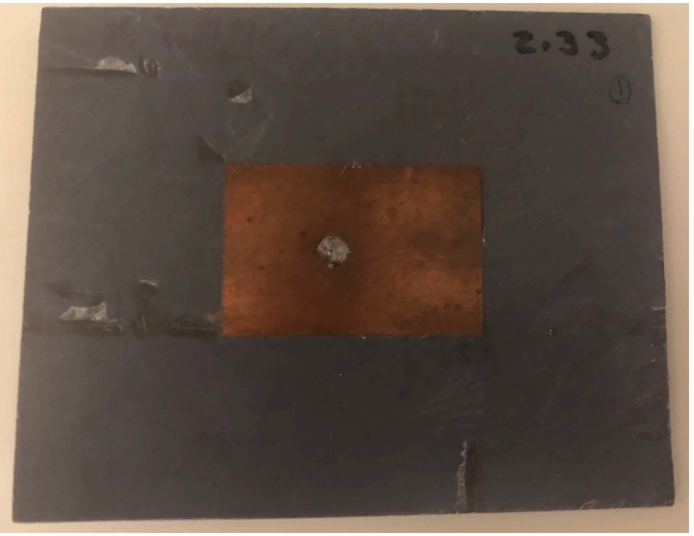
Cavity model with an effective dielectric constant: $f_r = \dfrac{c}{2\varepsilon_{reff} L}$

Cavity model with an effective length: $f_r = \dfrac{c}{2\varepsilon_r L_{eff}(\varepsilon_r)}$

Cavity model with an effective dielectric and length: $f_r = \dfrac{c}{2\varepsilon_{reff} L_{eff}(\varepsilon_{reff})}$

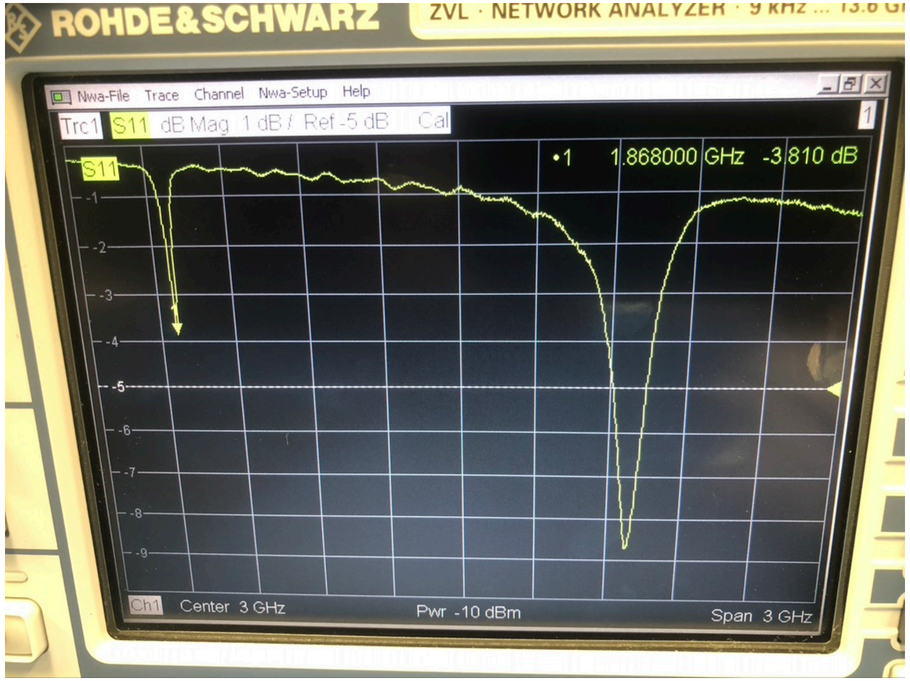
Adjustments

Five patch antennas were measured for the resonant frequency each of which has a fixed value of L. In some cases, the patch was constructed to be resonant along the shorter length. For these cases we define the shorter length as L and the longer length as W. For known values of L we can use the formulas to see if they correctly predict the measured resonance - verifying their accuracy.



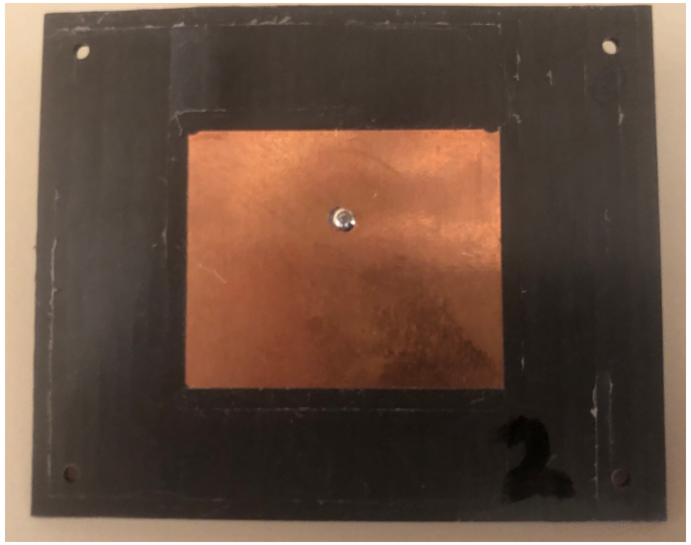
Antenna #1

Measured Resonance:
1.868 GHz



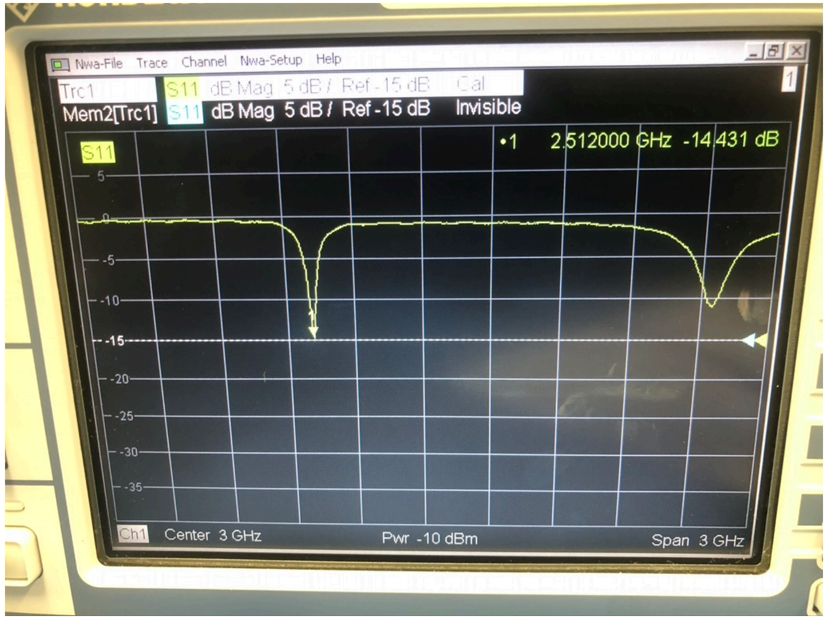
$$L = 5.13\ cm \quad W = 3.4\ cm \quad d = 0.1575\ cm \quad \varepsilon_r = 2.33$$

f_r	$\frac{c}{2\varepsilon_r L}$	$\frac{c}{2\varepsilon_{reff} L}$	$\frac{c}{2\varepsilon_r L_{eff}(\varepsilon_{reff})}$	$\frac{c}{2\varepsilon_r L_{eff}(\varepsilon_r)}$	$\frac{c}{2\varepsilon_{reff} L_{eff}(\varepsilon_{reff})}$
GHz	1.916	1.972	1.856	1.857	1.911



Antenna #2

Measured Resonance:
2.512 GHz

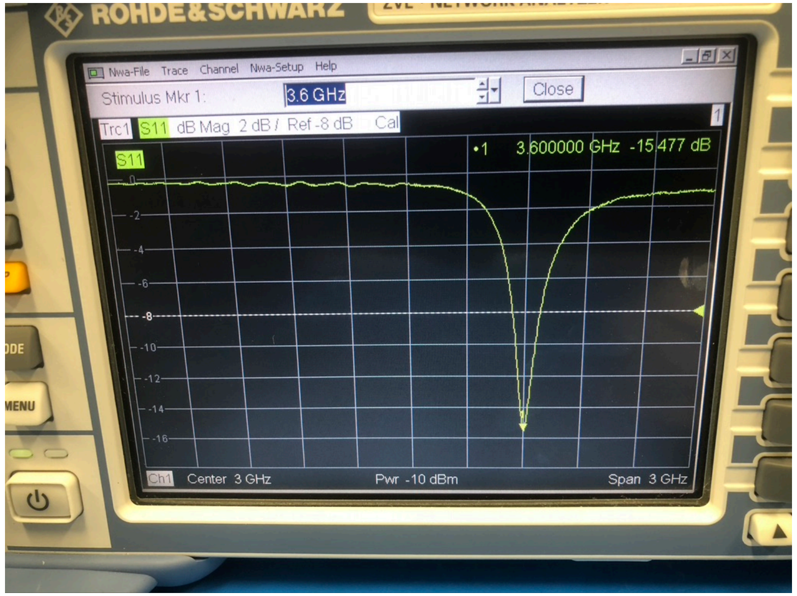


$$L = 4.0\ cm \quad W = 4.7\ cm \quad d = 0.1575\ cm \quad \varepsilon_r = 2.2$$

f_r	$\frac{c}{2\sqrt{\varepsilon_r}L}$	$\frac{c}{2\sqrt{\varepsilon_{reff}}L}$	$\frac{c}{2\sqrt{\varepsilon_r}L_{eff}(\varepsilon_{reff})}$	$\frac{c}{2\sqrt{\varepsilon_r}L_{eff}(\varepsilon_r)}$	$\frac{c}{2\sqrt{\varepsilon_{reff}}L_{eff}(\varepsilon_{reff})}$
GHz	2.528	2.584	2.427	2.429	2.481

Antenna #3

Measured Resonance:
3.60 GHz

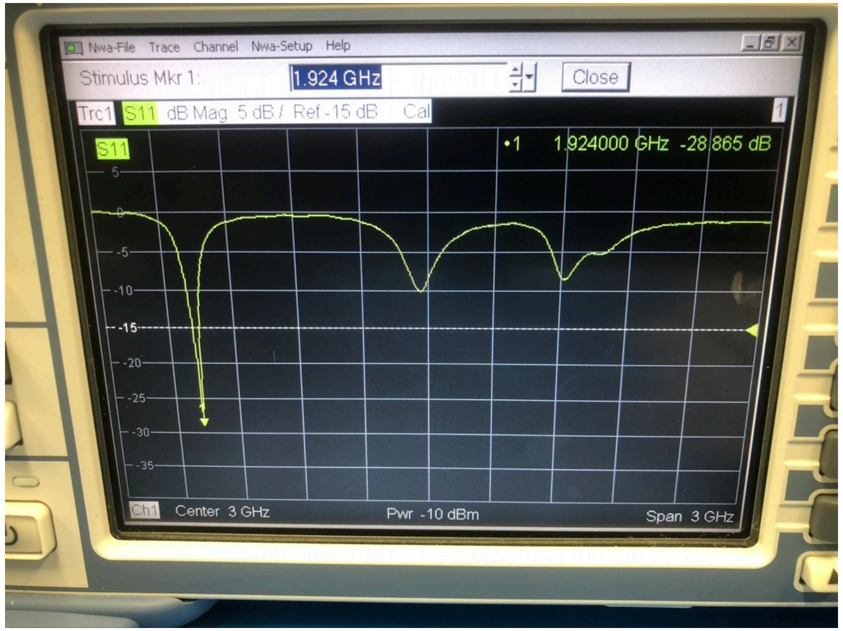


$$L = 1.95\ cm \quad W = 1.95\ cm \quad d = 0.1346\ cm \quad \varepsilon_r = 4.0$$

f_r	$\frac{c}{2\varepsilon_r L}$	$\frac{c}{2\varepsilon_{reff} L}$	$\frac{c}{2\varepsilon_r L_{eff}(\varepsilon_{reff})}$	$\frac{c}{2\varepsilon_r L_{eff}(\varepsilon_r)}$	$\frac{c}{2\varepsilon_{reff} L_{eff}(\varepsilon_{reff})}$
GHz	3.846	4.049	3.615	3.618	3.805

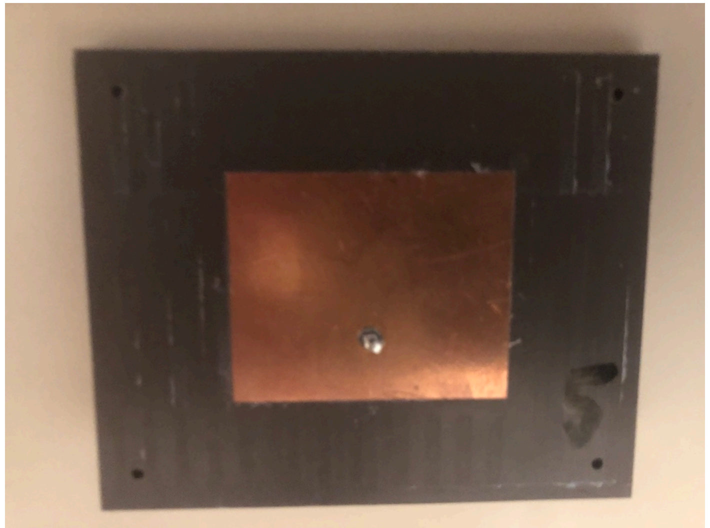
Antenna #4

Measured Resonance:
1.924 GHz

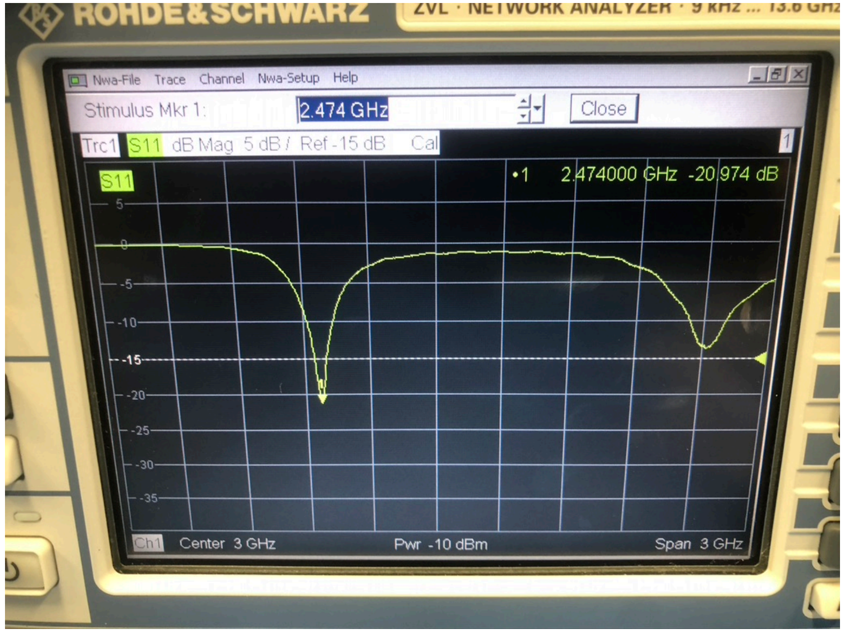


$$L = 4.9\ cm \quad W = 6.7\ cm \quad d = 0.254\ cm \quad \varepsilon_r = 2.2$$

f_r	$\frac{c}{2\varepsilon_r L}$	$\frac{c}{2\varepsilon_{reff} L}$	$\frac{c}{2\varepsilon_r L_{eff}(\varepsilon_{reff})}$	$\frac{c}{2\varepsilon_r L_{eff}(\varepsilon_r)}$	$\frac{c}{2\varepsilon_{reff} L_{eff}(\varepsilon_{reff})}$
GHz	2.064	2.114	1.957	1.958	2.004



Antenna #5



Measured Resonance:
2.474 GHz

$$L = 3.6\ cm \quad W = 4.4\ cm \quad d = 0.4826\ cm \quad \varepsilon_r = 2.2$$

f_r	$\frac{c}{2\varepsilon_r L}$	$\frac{c}{2\varepsilon_{reff} L}$	$\frac{c}{2\varepsilon_r L_{eff}(\varepsilon_{reff})}$	$\frac{c}{2\varepsilon_r L_{eff}(\varepsilon_r)}$	$\frac{c}{2\varepsilon_{reff} L_{eff}(\varepsilon_{reff})}$
GHz	2.809	2.951	2.468	2.476	2.592

Adjustments

The measured data and various calculations suggests that the effective dielectric constant ε_{reff} is not necessary for an accurate correction to the cavity model's resonant frequency. However, the effective length calculation is very important and is sufficient to correct the over estimate resonant frequency of the cavity model.