Teaching Directional Couplers and Directional Bridges

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Mo-A10-AM1-1

Directional Couplers and Directional Bridges

- Used as components of measurement chains to monitor forward and reflected power or as building blocks of measuring instruments, like VNAs
- These devices are usually presented in MD and PhD courses, at most, for the purpose of illustrating basic operation
- Metrological characteristics and limits of accuracy are rarely mentioned

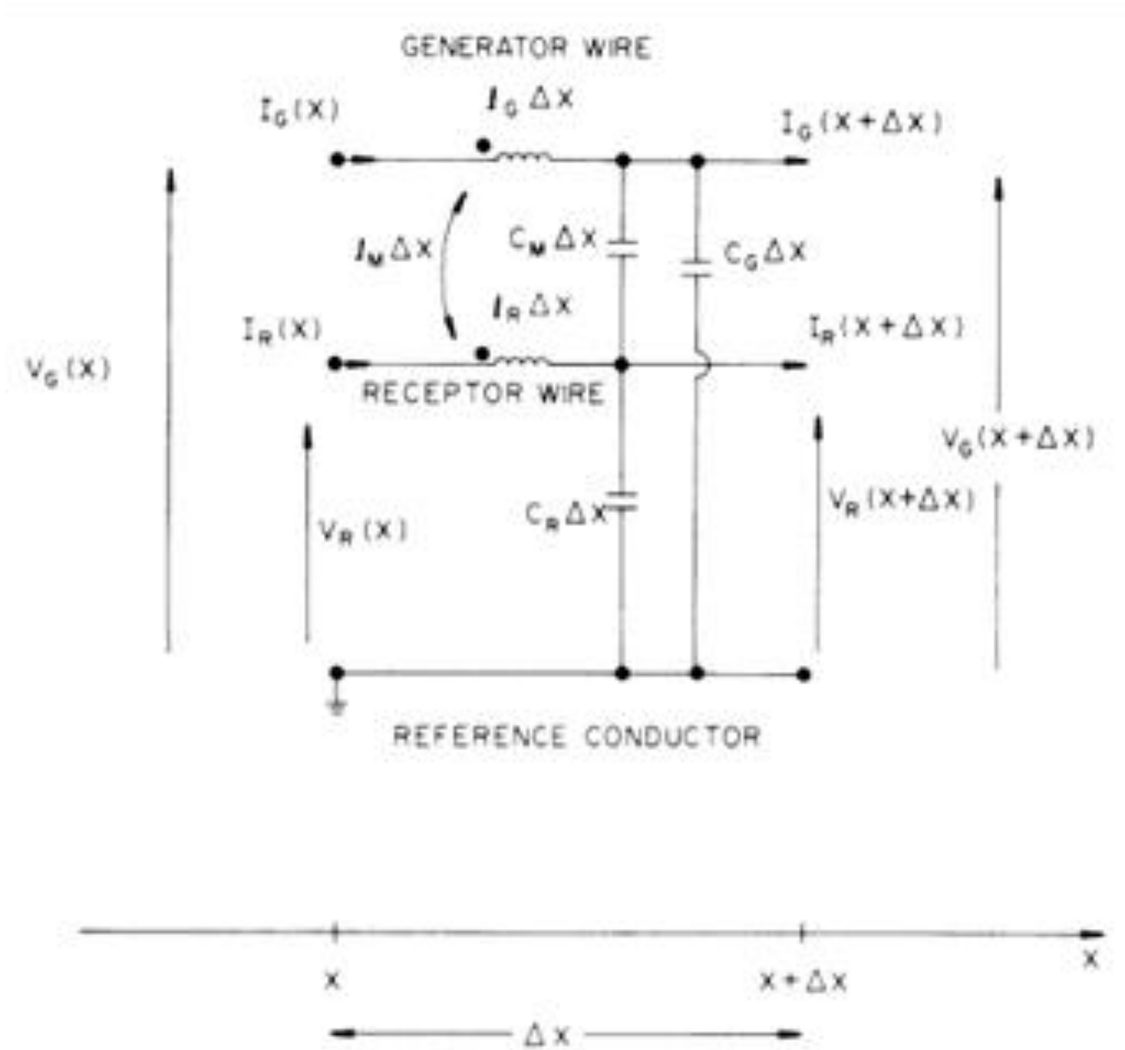
Scope

- Introduce the principle of operation of directional couplers and directional bridges
- Introduce essential metrological characteristics (coupling, isolation, directivity, insertion loss and mis-match)
- Provide equations useful to quantify errors in measurement of forward and reflected power (for professional use in calibration activity)

This contribution is in the vein of two others to URSI Com. A, Education and Training in Electromagnetic Technology:

- 1 - C. Carobbi, "Quantitative Experiments for Hands-on Training in RF and EMC Measurements," 32nd URSI GASS, Montreal, 19-26 August 2017.
- 2 - Carlo F. M. Carobbi, "Teaching radiofrequency power measurements," 33rd URSI GASS, Rome (virtual) 29 August – 5 September 2020.

Principle of operation of directional couplers



Multiconductor transmission line model (valid for coaxial and stripline couplers)

P.U.L. parameters, velocity of propagation, coupling factor, characteristic impedance

$$\mathbf{L} = \begin{bmatrix} l_G & l_m \\ l_m & l_R \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} c_G + c_m & -c_m \\ -c_m & c_R + c_m \end{bmatrix} \qquad \mathbf{LC} = \begin{bmatrix} 1/v^2 & 0 \\ 0 & 1/v^2 \end{bmatrix}$$

$$\begin{aligned} l_R &= l_G = l \\ c_R &= c_G = c \end{aligned} \qquad \begin{cases} l\left(c + c_m\right) - l_m c_m = \dfrac{1}{v^2} \\ -l c_m + l_m\left(c + c_m\right) = 0 \end{cases} \qquad \begin{cases} v = \dfrac{1}{\sqrt{1+k}} \dfrac{1}{\sqrt{lc}} \\ \dfrac{l_m}{l} = \dfrac{c_m}{c + c_m} \end{cases}$$

$$k = \frac{l_m}{l} = \frac{c_m}{c + c_m} \qquad Z_0 = \sqrt{\frac{l}{c + c_m}} = vl\sqrt{1 - k^2}$$

Solution of generalized Telegrapher's equations

$$\frac{d}{dx}\mathbf{V}(x) = -\mathbf{Z}\mathbf{I}(x) \quad \mathbf{V}(x) = \left[V_G(x), V_R(x)\right]^T \quad \mathbf{Z} = j\omega\mathbf{L}$$

$$\frac{d}{dx}\mathbf{I}(x) = -\mathbf{Y}\mathbf{V}(x) \quad \mathbf{I}(x) = \left[I_G(x), I_R(x)\right]^T \quad \mathbf{Y} = j\omega\mathbf{C}$$

$$V_G(0) = V_S/2$$

$$\frac{V_R(0)}{V_S/4} = \frac{\left[\frac{j\omega l_m \mathcal{L}}{Z_0} + j\omega c_m \mathcal{L} Z_0\right] \cdot \frac{\sin(\beta\mathcal{L})}{\beta\mathcal{L}}}{\cos(\beta\mathcal{L}) + j\frac{\sin(\beta\mathcal{L})}{\sqrt{1-k^2}}}$$

$$\frac{V_G(\mathcal{L})}{V_S/2} = \left[\cos(\beta\mathcal{L}) + j\frac{\sin(\beta\mathcal{L})}{\sqrt{1-k^2}}\right]^{-1}$$

$$\frac{V_R(\mathcal{L})}{V_S/4} = \frac{\left[-\frac{j\omega l_m \mathcal{L}}{Z_0} + j\omega c_m \mathcal{L} Z_0\right] \cdot \frac{\sin(\beta\mathcal{L})}{\beta\mathcal{L}}}{\left(\cos(\beta\mathcal{L}) + j\frac{\sin(\beta\mathcal{L})}{\sqrt{1-k^2}}\right)^2}$$

V_S is the open circuit voltage of the source, lines terminated by the characteristic impedance

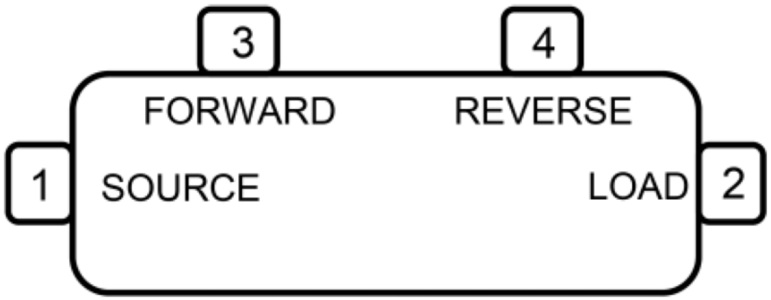
The directional coupler is served!

$$\frac{l_m}{c_m} = Z_0^2 \quad \Rightarrow \quad V_R(0) = j\frac{V_S}{2}\sin\left(\beta\boldsymbol{\mathcal{L}}\right)\frac{k/\sqrt{1-k^2}}{\cos\left(\beta\boldsymbol{\mathcal{L}}\right)+j\frac{\sin\left(\beta\boldsymbol{\mathcal{L}}\right)}{\sqrt{1-k^2}}}$$

$$V_R(\boldsymbol{\mathcal{L}}) = 0$$

The interpretation is immediate in the limit as $\beta\mathcal{L} \to 0$ (lumped model). In particular, we see that $V_R(\mathcal{L}) = 0$ because the effects of magnetic (inductive) and electric (capacitive) coupling cancel out at $x = \mathcal{L}$, while add in phase at $x = 0$. Due to imperfections (asymmetries) in the practical realization of the directional coupler this cancellation cannot be total.

Scattering matrix (ideal device)



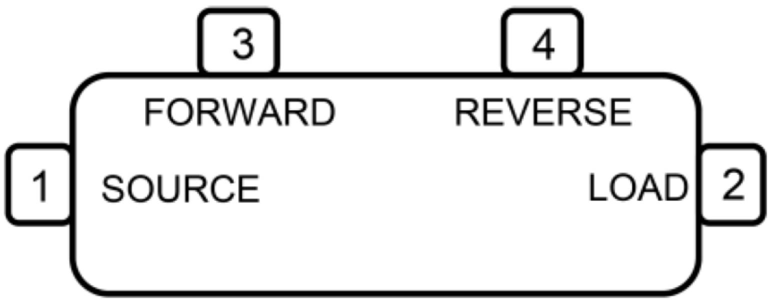
$$\mathbf{S} = \begin{pmatrix} 0 & S_{21} & S_{31} & 0 \\ S_{21} & 0 & 0 & S_{31} \\ S_{31} & 0 & 0 & S_{21} \\ 0 & S_{31} & S_{21} & 0 \end{pmatrix}$$

$$\mathcal{L} = \lambda / 4 \Rightarrow$$

$$S_{31} = k$$

$$S_{21} = -j\sqrt{1-k^2}$$

Insertion loss, coupling, isolation, directivity, return loss (real device)



$$IL = -20\log\left|S_{21}\right|$$

$$C = -20\log\left|S_{31}\right|$$

$$I = -20\log\left|S_{41}\right|$$

$$D = -20\log\frac{\left|S_{41}\right|}{\left|S_{31}\right|}$$

$$\Rightarrow D = I - C$$

$$RL = -20\log\left|S_{22}\right|$$

Complex reflection coefficient measurement model

$$\Gamma = \Lambda \frac{\Gamma' - \frac{1}{\Delta}}{1 - \frac{\Lambda}{\Delta}\Sigma + \Gamma'\left(\Lambda\Sigma - \frac{1}{\Delta}\right)}$$

$$\Delta = S_{31}/S_{41}$$

$$\Sigma = S_{22}$$

$$\Lambda = 1/S_{21}$$

Γ is the true complex reflection coefficient (reflection coefficient of the load @ port 2)
Γ' measured complex reflection coefficient (ratio of waves impinging to matched loads @ port 4 and @ port 3)

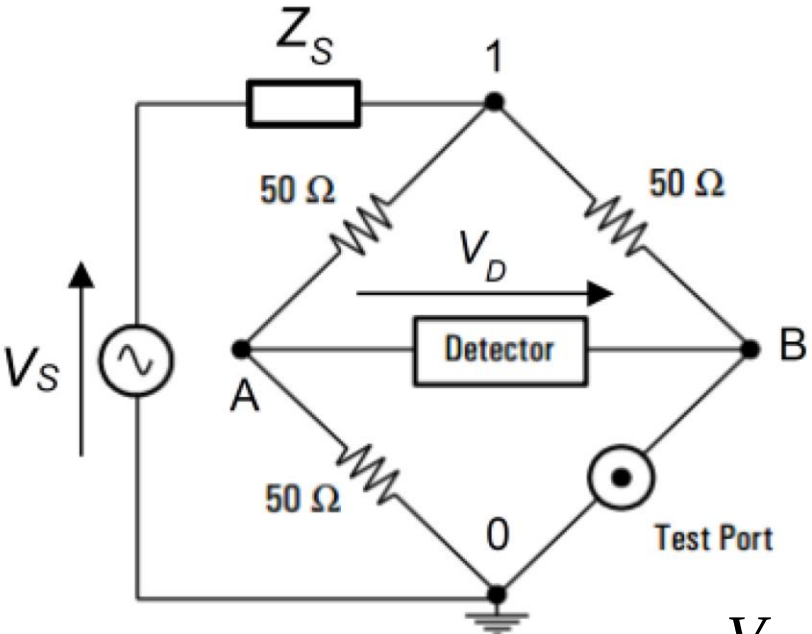
For a quasi-ideal coupler $|\Delta| \to \infty$ and $|\Sigma| \to 0$ then the following expression is derived which leads to an immediate interpretation

$$\Gamma \to \Lambda\left[\Gamma' - \frac{1}{D} - \left(\Gamma'\right)^2 \Lambda\Sigma\right]$$

Directional bridge

- Similarly to a directional coupler permits to discriminate forward and reflected waves (reflectometer).
- A wide-band reflectometer cannot be realized by a directional coupler whose bandwidth is of few octaves.
- A single directional bridge can instead cover several frequency decades, from few kilohertz up to several gigahertz.
- The use of directional bridges is more convenient at low frequency, while directional couplers are more suitable at high frequency, starting from several hundreds of megahertz to microwaves.

Principle of operation of directional bridge



Z_X unknown test impedance

$Z_0 = 50\,\Omega$

$$\frac{V_D}{V_D\big|_{Z_X=\infty}} = \frac{(Z_X - Z_0)(3Z_S + 5Z_0)}{Z_X(3Z_S + 5Z_0) + Z_0(5Z_S + 3Z_0)}$$

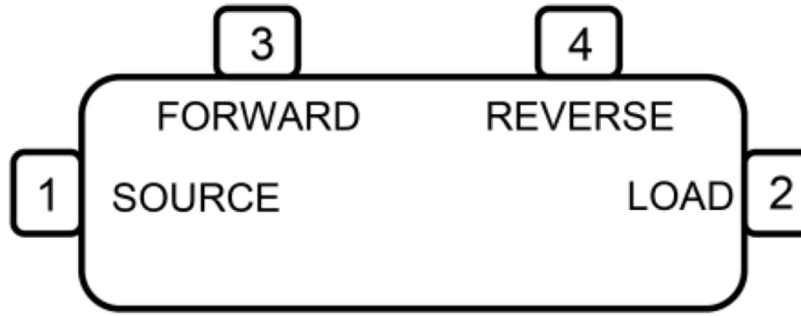
$Z_S = Z_0$ ⇒ $$\frac{V_D}{V_D\big|_{Z_X=\infty}} = \frac{Z_X - Z_0}{Z_X + Z_0} = \Gamma$$

Single-ended source and detector

- To prevent the test port arm of the directional bridge from being short-circuited by a single ended detector two solutions are possible:
  - Voltage between node 1 and node 0 is provided by the secondary of a 1:1 radiofrequency transformer whose primary is fed by the source.
  - Symmetrically tapping the voltage across the bridge diagonal with respect to ground through an identical couple of inductive chokes.
- In both cases a low-frequency limit (corner frequency) is introduced.
- Asymmetries in the 1:1 transformer or in the chokes degrade the performance of the directional bridge in terms of directivity.

Directional bridge seen as a directional coupler

- Port 3 internally terminated
- Remaining accessible ports are port 1 (Source), port 2 (or Test Port, Load), port 4 (Detector, Reverse)
- Scattering matrix of the ideal component



$$\mathbf{S} = \begin{pmatrix} 0 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 0 \end{pmatrix}$$

Complex reflection coefficient measurement model (real device)

$$\Gamma = \frac{\Gamma'\left(\frac{1}{\Delta} + \frac{1}{1-\Sigma}\right) - \frac{1}{\Delta}}{1 + \Sigma\left[\Gamma'\left(\frac{1}{\Delta} + \frac{1}{1-\Sigma}\right) - \frac{1}{\Delta}\right]}$$

Γ' is the ratio between the forward waves to the detector in case of load/no-load ($Z_X = \infty$)

$\Delta = S_{21}S_{32}/S_{31}$

$\Sigma = S_{22}$

Δ corresponds to directivity (isolation/coupling), Σ corresponds to return loss

For a quasi-ideal directional bridge $|\Delta| \to \infty$ and $|\Sigma| \to 0$ then the following expression is derived which leads to an immediate interpretation

$$\Gamma \to \Gamma' - \frac{1}{D} - (\Gamma')^2\,\Sigma$$

Conclusion

- When teaching a measurement device care should be given to
  - Analyze its non-ideal behavior and explain its physical origin
  - Derive modelling equations to calculate the best estimate of the measurand and its uncertainty
- Results obtained for directional couplers and bridges involve non-trivial complex valued models, useful to introduce VNA measurement error analysis

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